Onset of an Insulating Zero-Plateau Quantum Hall State in Graphene

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We analyze the dissipative conductance of the zero-plateau quantum Hall state appearing in undoped graphene in strong magnetic fields. Charge transport in this state is assumed to be carried by a magnetic domain wall, which forms by hybridization of two counter–propagating edge states of opposing spin due to interactions. The resulting non–chiral edge mode is a Luttinger liquid of parameter K, which enters a gapped, perfectly conducting state below a critical value $K_c \approx 1/2$. Backscattering in this system involves spin flip, so that interaction with localized magnetic moments generates a finite resistivity R_{xx} via a "chiral Kondo effect". At finite temperatures T, $R_{xx}(T)$ exhibits a crossover from metallic to insulating behavior as K is tuned across a threshold K_{MI}. For $T \to 0$, R_{xx} in the intermediate regime $K_{MI} < K < K_c$ is finite, but diverges as K approaches K_c. This model provides a natural interpretation of recent experiments.

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Introduction– Graphene, a honeycomb network of carbon atoms, is perhaps the most remarkable two-dimensional system to be studied in the last few years [1]. This system differs from the standard two dimensional electron gas (2DEG) in supporting two Dirac points in its fermion spectrum which are at the Fermi energy when the system is nominally undoped. In the presence of a magnetic field, the Landau level spectrum differs from the standard 2DEG in supporting positive and negative energy (Landau level) states, as well as a zero energy (lowest) Landau level in each of its two valleys. In the earliest experiments, this last property was understood to account for the absence of a quantized Hall effect at filling factor $\nu = 0$ [2, 3].

In stronger fields, and with higher quality samples, a plateau does appear to emerge near $\nu = 0$ which is likely associated with a resolution of the spin-splitting of the lowest Landau level [4, 5, 6]. However, the behavior of the longitudinal (dissipative) resistance (R_{xx}) at this filling has been difficult to explain: different measurements have demonstrated that it may either decrease [5] or increase [6] with falling temperature T in samples that appear to be quite similar [7]. Moreover, in Ref. [6] the $\nu = 0$ quantized Hall state exhibits a transition to an insulator at a critical magnetic field H_c, where $R_{xx}(T \to 0)$ diverges. The scaling of R_{xx} as a function of field H below H_c appears to signify a quantum phase transition of the Kosterlitz–Thouless (KT)[8] type.

In this paper, we demonstrate that such a variety of transport behaviors can be explained quite naturally if the system contains local magnetic moments near its edge, which may occur due to chemical passivation of dangling bonds or imperfections in the lattice [9]. Interaction with such magnetic degrees of freedom provides a mechanism for back–scattering in the primary channel

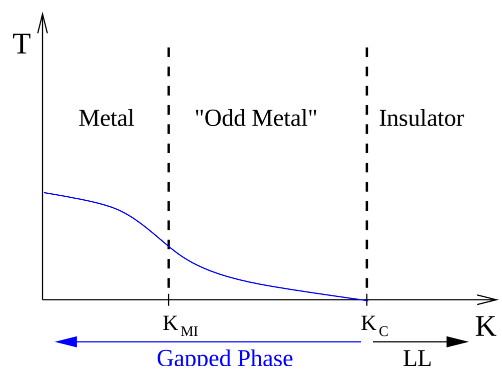

FIG. 1: (Color online). Phase diagram of the DW coupled to a magnetic impurity via a fixed Kondo interaction. The solid blue curve depicts the gap $\Delta_s(K)$. The dashed lines mark the boundaries between distinct transport regimes (see text).

for charge conduction at $\nu = 0$, a magnetic domain wall (DW) [10, 11, 12] which may carry current that is unaffected by ordinary, *non–magnetic* impurities. Transport properties of this mode are dramatically altered by tuning of a Luttinger parameter K [13], which in this system depends on the details of the edge potential and on the magnetic field H.

Our main findings are summarized in the phase diagram depicted in Fig. 1. For $K > K_c$, the ideal system is a Luttinger liquid, with K large enough that a Kondo impurity renders the system insulating at zero temperature [$R_{xx} \to \infty$]. As K is tuned below K_c, the impurity-free DW system undergoes a quantum phase transition to a perfect conducting state protected by a gap $\Delta_s(K)$ (solid line in Fig. 1). In this case a Kondo impurity generates a *finite* resistance at zero temperature, approaching this value from *below* ($dR_{xx}/dT < 0$). The unusual thermal behavior of this "odd metal," results from a competi-

tion between two relevant perturbations which separately would render the system either perfectly conducting or truly insulating. Finally, for $K < K_{MI}$, a Kondo impurity is an irrelevant perturbation, yielding more typical metallic behavior, $dR_{xx}/dT > 0$. This variety of behavior is consistent with existing published data [5, 6].

The remarkable edge–mode structure of the DW can already be seen within a non-interacting picture. The lowest Landau levels support an electron-like and a hole-like mode for each spin, which cross precisely at the Fermi level when Zeeman splitting is introduced [see Fig. 2] [10, 11, 14]. Accounting for interactions within a Hartree-Fock description, the counter–propagating states admix to form the DW [12], with an unusual collective mode. In particular, due to the spin–charge coupling inherent in a state projected into a single Landau level [15], it supports topological *charged* excitations which can propagate freely along the edge. In addition, a spin gap may open in the collective excitation spectrum when interactions are not too strong, with important consequences for dissipative transport.

A prominent candidate for a dissipation mechanism in this system is the scattering between counter–propagating edge states [16], which one might expect to be enhanced by the DW structure which strongly hybridizes forward and backward propagating states. However, the requirement of spin conservation forbids backscattering within a single DW by static impurities [11]. In contrast, scattering from local *magnetic* impurities *does* allow backscattering and generates dissipation. As we show below, the coupling of a DW to a magnetic impurity leads to an unusual "chiral" Kondo effect, since left and right moving electrons each come in only a single spin flavor. The resulting resistance at finite T exhibits the variety of behaviors summarized above, which we discuss below in more detail.

Model – As a first stage we derive an effective model for a clean graphene sheet, with a straight edge along the y direction. Our starting point for the analysis is to view the system as a two-dimensional (2D) quantum Hall ferromagnet, in which only the two Landau levels closest to zero energy are included. These levels carry opposite spin, and may be grouped together into a spinor whose long-wavelength degrees of freedom are those of a Heisenberg ferromagnet, with Hamiltonian [12, 15]

$$H^0_{2D} = \int d^2r \left\{ \frac{J}{2} \sum_\mu |\partial_\mu \mathbf{S}(\mathbf{r})|^2 + \Delta(x) S_z(\mathbf{r}) \right\} . \quad (1)$$

Here J represents an exchange stiffness due to interactions. $\Delta(x)$ encodes the combined effects of the Zeeman interaction and the edge potential (dictated by the boundary conditions and electrostatic environment [17]): it increases monotonically from the bulk (negative) Zeeman energy for x well inside the bulk to a positive value at the edge, leading to the energy cross-

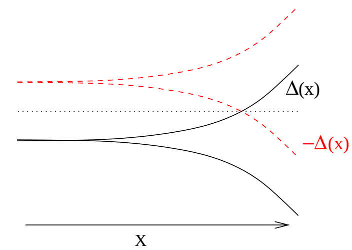

FIG. 2: (Color online). Non–interacting energy levels near the edge. Solid black lines are spin up states, dashed red lines are spin down states. The dotted line denotes the chemical potential at zero energy.

ing in the non-interacting spectrum illustrated in Fig. 2. Initially treating the system classically, we parameterize the spin field by $\mathbf{S}(\mathbf{r}) = S\boldsymbol{\Omega}(\mathbf{r})$, with $S = 1/2$ and $\boldsymbol{\Omega} = [\sin\psi(\mathbf{r})\cos\chi(\mathbf{r}), \sin\psi(\mathbf{r})\sin\chi(\mathbf{r}), \cos\psi(\mathbf{r})]$. As shown in [12], the form of $\Delta(x)$ in the above energy functional dictates a minimum energy configuration where χ is spatially constant, and $\psi(\mathbf{r}) = \psi_0(x)$ exhibits a non-trivial topology: $\psi_0(x \to -\infty) \to 0$, $\psi_0(x \to \text{edge}) \to \pi$. A domain wall (DW) is thus formed parallel to the edge. The ground state energy is independent of χ, implying a broken symmetry and an associated collective mode propagating with a wavevector k_y along the DW [18].

The 2D semiclassical theory may be projected into its low-energy subspace, containing the one-dimensional (1D) mode. This theory is identical in form to a semiclassical treatment of an XXZ antiferromagnetic spin-1/2 chain; an appropriate choice of coefficients for the latter allows the models to coincide at the semiclassical level. As the spin chain incorporates the spin-1/2 nature of the system, we work with this model, with lattice spacing set by the magnetic length $\ell = \sqrt{\hbar c/eH}$:

$$H_{DW} = J_{xy} \sum_j \left[S^x_j S^x_{j+1} + S^y_j S^y_{j+1} \right] + J_z \sum_j S^z_j S^z_{j+1} \, . \quad (2)$$

Here J_{xy} is related to the exchange energy J via $J_{xy} = JS^2\mathcal{N}$, with $\mathcal{N} = \sum_x \sin^2\psi_0(x)$ ($\psi_0(x)$ the classical DW texture), and J_z depends on the edge potential: $J_z = \frac{1}{\mathcal{N}2S}\sum_x \Delta(x)\sin^2\psi_0(x)[1 - \cos\psi_0(x)]$. Using a standard Bosonization scheme for the spin operators in the continuum limit [13],

$$S_\pm(y) = \frac{e^{\mp i\theta}}{\sqrt{2\pi\alpha}}[(-)^y + \cos(2\phi)]$$
$$S_z(y) = -\frac{\partial_y\phi}{\pi} + \frac{(-)^y}{\pi\alpha}\cos(2\phi) \, , \quad (3)$$

we map H_{DW} to a (single–flavored) Luttinger model with a sine–Gordon correction. In units where $\hbar = 1$,

$$H_{DW} = \int \frac{dy}{2\pi} \left\{ uK(\pi\Pi)^2 + \frac{u}{K}(\partial_y\phi)^2 - \frac{J_z}{\pi\alpha^2}\cos(4\phi) \right\} \quad (4)$$

where the Boson field $\phi(y)$ and $\Pi(y) = \frac{\partial_y \theta}{\pi}$ are canonically conjugate. The spin–wave velocity u and Luttinger parameter K are related to the XXZ parameters by

$$uK = J_{xy} \ , \quad \frac{u}{K} = J_{xy} + \frac{4J_z}{\pi} \ , \tag{5}$$

and the short distance cutoff $\alpha \sim \ell$.

The effective model for the ideal DW system [Eq. (4)] exhibits a quantum phase transition upon tuning of the parameter K across a critical point K_c, from a Luttinger liquid (LL) phase at $K > K_c$ [where the $\cos(4\phi)$ term is irrelevant] to an ordered [gapped] phase below K_c. In terms of the spin fields, the latter phase is characterized by ordering of the field S_z, and the opening of a gap Δ_s to spin-flip excitations. The critical behavior near K_c can be derived from a perturbative renormalization group (RG) analysis [13], yielding a KT–transition at $K_c \sim 1/2$. As K approaches K_c from below, the gap tends to vanish according to the scaling law

$$\Delta_s(K) \simeq \frac{u}{\alpha} \exp\left\{-\frac{C}{\sqrt{K_c - K}}\right\} \tag{6}$$

with C a constant of order unity [see Fig. 1].

We now assume that a local moment at the origin (represented by a spin-1/2 operator $\vec{\sigma}$) couples to the original 2D spin system via an exchange interaction $\tilde{J}\vec{S}(\mathbf{r}=0)\cdot\vec{\sigma}$. In order to project this into the low–energy subspace, we express it in a rotated basis where the local moment is coupled to fluctuations about the ground state DW configuration. This yields an effective Kondo coupling to the 1D spin chain operators:

$$H_K = \frac{J_K^{xy}}{2}(S_+(0)\sigma_- + S_-(0)\sigma_+) + J_K^z S_z(0)\sigma_z \ . \tag{7}$$

The resulting Hamiltonian for the effective 1D system then takes the form $H = H_0 + H_K$, with

$$H_0 = H_{DW} + \tilde{E}_z\sigma_z \ . \tag{8}$$

Note that $\tilde{E}_z$ is an effective Zeeman energy of the localized spin, associated with a *local* magnetic field which depends on its position within the width of the DW.

Transport – To derive the transport properties of the system, we first express the charge current operator in terms of the DW fields. Using the standard definition $j_e = -\frac{\delta H_{DW}}{\delta A}$, with $A(y,t)$ a vector potential, we find

$$j_e(y,t) = 2eJ_z S_z(y,t) \ . \tag{9}$$

This remarkable relation between spin and charge current encodes a unique property of the DW: in the absence of spin-flip processes (i.e. for $J_K^{xy} = 0$), current is conserved and the DW behaves as a perfect conducting channel. The distinction between its two phases becomes apparent when spin impurities are added: in the gapped phase, spin-flip and hence back-scattering excitations are suppressed for $T < \Delta_s$. Its 'perfect conduction' is hence more robust. Indeed, H_{DW} [Eq. (4)] is analogous to a low–energy model of a quasi 1D *superconducting* system, where two parallel wires are Josephson coupled. The field ϕ describes the relative superconducting phase, u/K the superfluid stiffness, uK the charging energy [19], and the last term in H_{DW} is the inter-wire Josephson coupling. There as well, a local perturbation analogous to H_K [Eq. (7)] is required to generate finite dissipation via a phase–slip mechanism. This occurs when a Josephson vortex (i.e., out-of-phase current configuration) penetrates into the bulk of the double-wire system. In the gapped phase, where Josephson coupling is relevant, such processes are suppressed beyond the Josephson length ($\sim 1/\Delta_s$).

We next consider a finite but small Kondo interaction and $T > \Delta_s$, and calculate the dissipative part of the electric conductance to leading order in H_K:

$$\delta G = \lim_{\omega\to 0} \frac{-1}{L\omega} \int_{-L/2}^{L/2} dy \Im\{\chi(y, y'; \omega)\} \ , \tag{10}$$

where $\chi = \langle j_e(y); j_e(y')\rangle^0_\omega$ is the retarded correlation function evaluated with respect to H_0 (see, e.g., [13]), and L the length of the system, assumed finite. Note that δG is a *negative* correction to the ideal conductance, so that $|\delta G|$ is proportional to the backscattering rate and consequently to the longitudinal resistance R_{xx} [20]. Using the bosonized representation Eqs. (3),(4) and the expression for the local spin correlator (for $t > 0$)

$$\langle\sigma_+(t)\sigma_-(0)\rangle^0 = e^{i\tilde{E}_z t} f\left(\frac{\tilde{E}_z}{T}\right) \ , \quad f(z) \equiv \frac{1}{e^z + 1} \tag{11}$$

we obtain the T–dependent conductance

$$\delta G(T) \approx -\frac{e^2}{h} g_K^2 \left(\frac{\pi\alpha T}{u}\right)^{\kappa-2} \mathcal{G}\left(\frac{\tilde{E}_z}{T}, \frac{\alpha T}{u}\right) \ . \tag{12}$$

Here $g_K \propto J_K^{xy}$ is dimensionless, $\kappa = 1/2K$ and

$$\begin{aligned}\mathcal{G}(z,\epsilon) \ &\equiv \ \sin\left(\frac{\pi\kappa}{2}\right) \int_\epsilon^\infty dt \frac{t\cos(zt)}{[\sinh(\pi t)]^\kappa} \\ &- \ 2m(z)\cos\left(\frac{\pi\kappa}{2}\right) \int_\epsilon^\infty dt \frac{t\sin(zt)}{[\sinh(\pi t)]^\kappa}\end{aligned} \tag{13}$$

with $m(z) = [f(z) - f(-z)]$ the local spin magnetization.

In the finite T regime where $T \gg \tilde{E}_z$, the leading T–dependence of $\delta G(T)$ inferred from Eqs. (12), (13) is a power–law, which changes sign at $\kappa = 2$:

$$\delta G(T) \sim -T^{\kappa-2} \ . \tag{14}$$

The conductance therefore exhibits a crossover from a metallic behavior ($dG/dT < 0$) for $\kappa > 2$ to an insulating behavior ($dG/dT > 0$) for $\kappa < 2$. This crossover would be a true metal–insulator (MI) as K is tuned through

$K_{MI} \approx 1/4$, as depicted in Fig. (1), if not for the existence of the gap. Note that K_{MI} is perturbatively renormalized to values below $1/4$ by J_K^{xy} and J_K^z, so that one always finds $K_{MI} < K_c$. The MI crossover then always occurs within the gapped phase. Thus the power–law behavior of $G(T)$ is restricted to finite $T \gg \Delta_s$.

We now focus on the intermediate regime $K_{MI} < K < K_c$. Assuming further that $\tilde{E}_z \sim 0$, we consider the low T limit $T \ll \Delta_s$ where the field ϕ is ordered, while θ is strongly fluctuating. The power–law decay of the correlation function $\langle e^{i\theta(t)} e^{-i\theta(0)} \rangle^0$ (characteristic of the LL) is modified by a rapidly oscillating exponential factor:

$$\langle e^{i\theta(t)} e^{-i\theta(0)} \rangle^0 \approx \left(\frac{\alpha}{ut}\right)^{\kappa} \exp\left\{-\frac{\kappa \Delta_s t^2}{\tau_\phi} + i\kappa \left(\frac{\pi}{2} + \frac{t}{\tau_\phi}\right)\right\} \tag{15}$$

where the decoherence rate $\tau_\phi^{-1} \equiv \Delta_s^2 L / 2\pi u$ corresponds to a 'condensation energy' in the superconductor analogue. The resulting δG saturates as $T \to 0$ and becomes

$$\delta G \sim -\tau_\phi^{2-\kappa} \, . \tag{16}$$

It then follows from the definition of τ_ϕ and Eq. (6) that the resistance *diverges* when K approaches K_c as

$$R_{xx} \sim -\delta G \sim \exp\left\{\frac{2C'}{\sqrt{K_c - K}}\right\} \, . \tag{17}$$

Here $C' = C(2-\kappa)$; for $\kappa \sim 1/2K_c \sim 1$, $C' \sim C$.

We finally comment on the possible relation of our results to the experimentally observed R_{xx} in the zero–plateau state. Provided K is tunable by a magnetic field H, the system is shown to exhibit diverse transport properties, which could explain the apparent discrepancy between the data of Refs. [5] and [6]. Furthermore, we find a quantum critical point associated with the closing of a spin–gap, manifested by a divergence of $R_{xx}(T \to 0)$ according to a scaling law characteristic of a KT–transition (in $1+1$–dimensions). The latter is remarkably reminiscent of the peculiar field-tuned transition to an insulator observed in [6]. A rough estimate of K in terms of physical parameters yields $K \sim \sqrt{\frac{e^2}{\epsilon \ell \Delta_{av}}}$, with Δ_{av} an average edge potential strongly dependent on sample details. In particular, when Δ_{av} is dominated by electrostatic effects induced by the gate voltage [17], it is typically larger than the exchange energy $\frac{e^2}{\epsilon \ell}$. This would yield $K < 1$ and monotonically increasing with H, as assumed by our theory.

Another assumption of our theory which requires justification is the neglect of $\tilde{E}_z$. A finite $\tilde{E}_z$ introduces a cutoff which competes with the gap Δ_s, and a finite magnetization of the local moment which reduces its contribution to backscattering. We note, however, that the realistic system presumably contains a multitude of spin impurities with randomly distributed local parameters. The dissipative resistance is then dominated by the moments with minimal $\tilde{E}_z$, and our analysis applies as long as it is smaller than all other energy scales.

To summarize, we study a model for charge transport carried by fluctuations of a DW propagating along the edge of a graphene sample in the $\nu = 0$ quantized Hall state. Backscattering induced by localized magnetic impurities at the edge provides a dissipation mechanism. Its competition with a spin–gap generated in the ideal DW may possibly explain the rich variety of conductance properties observed in recent experiments.

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